Probing deviations from tri-bimaximal mixing through ultra high energy neutrino signals

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Abstract

We investigate deviation from the tri-bimaximal mixing in the case of ultra high energy neutrino using ICECUBE detector. We consider the ratio of No. of muon tracks to the shower generated due to eletrons and hadrons. Our analysis shows for tri-bimaximal mixing the ratio comes out around 4.05. Keeping $\theta_1 2$ and $\theta_2 3$ fixed at tri-bimaximal value, we have varied the angle $\theta_{13} = 3^o$, 6^o, 9^o and the value of the ratio gradually decreases. The variation of ratio lies 8% to 18% from the tri-bimaximal mixing value.

1 Introduction

Various experiments for solar and atmospheric neutrinos provide a range for the values of solar mixing angle $\theta_{\odot} = \theta_{21}$ (the 2-1 mixing angle) angle [1] that corresponds to solar neutrino oscillations and also a range for atmospheric mixing angle $\theta_{\rm atm} = \theta_{23}$ (the 2-3 mixing angle) [1] around their best fit values. The tri-bimaximal mixing condition of neutrinos are given by $\sin\theta_{12} = \frac{1}{\sqrt{3}}$, $\sin\theta_{23} = \frac{1}{\sqrt{2}}$ and $\sin\theta_{13} = 0$ [2]. Possible deviations from tri-bimaximal mixing can be obtained by probing the ranges of θ_{12} and θ_{23} given by the experiments. Also the exact 13 mixing angle θ_{13} is not known except that the CHOOZ [3] gives an upper limit for $\theta_{13}(< 9^o)$. Probing the deviations of θ_{21} and θ_{23} for different values for θ_{13} is significant not only to understand the neutrino flavour oscillations in general but also for the purpose of model building for neutrino mass matrices.

In this work we explore the possibility for ultra high energy (UHE) neutrinos from distant Gamma Ray Bursts (GRBs) for probing the signatures of these deviations of the values of the mixing angles from tri-bimaximal mixing as discussed above. One such proposition of using UHE neutrinos is described in a recent work by Xing [4]. Gamma Ray Bursts are short lived but intense burst of gamma rays. During its occurance it outshines all other luminous objects in the sky. Although the exact machanism of GRBs could not be ascertained so far but the general wisdom is that it is powered by a central engine provided by a failed star or supernova that possibly turned into a black hole, accretes mass at its surroundings. This infalling mass due to gravity bounces back from the surface of black hole much the same way as the supernova explosion mechanism and a shock is generated that flows radially outwards with enormous amount of energies ($\sim 10^{53}$ ergs). This highly energetic shock wave drives the mass outwards, in the form of a "fireball" that carries in it, protons, γ etc. The pions are produced when the accelerated protons inside the fireball interacts with γ through a cosmic beam dump process. UHE neutrinos are produced by the decay of these pions. Thus a generic cosmic accelerator accelerates the protons into very high energies which then beam dumps on γ in the "fireball" as also at the cosmic microwave background (CMB) and ultra high energy neutrinos are produced.

The GRB neutrinos, due to their origin at astronomical distances from earth, provide a very long baseline for the earth bound detectors for UHE neutrinos such as ICECUBE [5]. The oscillatory part of the neutrino flavour oscillation probabilities ($\sin^2(\Delta m^2[L/4E])$) averages out to 1/2 becuase of this very long baseline L ($\sim$ hundreds of Mpc) and the Δm^2 (mass square difference of two neutrinos) range obtained from solar and atmospheric neutrino experiments are $\Delta m^2_{21} \sim 10^-4 eV^2$ and $\Delta m^2_{32} \sim 10^-3 eV^2$ respectively ($L/\Delta m^2 >> 1$). Thus for neutrino flavour oscillation, in this case, the effect of Δm^2 is washed out and governed only by the three mixing angles namely $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_{\rm atm}$ and θ_{13}. The purpose of the present work is to probe whether or not the possible variations of θ_{12} and θ_{23} from their best fit values can be ascertained by UHE from distant GRBs.

The GRB neutrinos, on arriving the earth, undergo charged current (CC) and neutral current (NC) interactions with the earth rock and the detector material. The CC interactions of ν_μ produce secondary muons and the same for electrons produce electromagnetic shower ($\nu_\mu + N \to \mu + X$ and $\nu_e + N \to e + X$). The former will produce secondary muon tracks and can be detected by track-signal produced by the Cerenkov light emitted by these muons during their passage through a large underground water/ice Cerenkov detectors like ICECUBE. The ICECUBE is a $1 {\rm km}^3$ detector in south pole ice and can be considered to be immersed in the target material for the UHE neutrinos where the neutrino interactions are initiated. In case of ν_e, the electrons from the $\nu_e N$ CC interactions, shower quickly and can also be detected by such ICECUBE detector. The case of ν_τ is somewhat complicated. The first CC interaction of ν_τ ($\nu_\tau + N \to \tau + X$) produces a shower ("first bang") alongwith a τ track. But the ν_τ is regenerated (with diminished energy) by the decay of τ and in the process produces another hadronic or electromagnetic shower ("second bang"). The whole process is called double bang event. In case the first bang could not be detected, then by possible detection of second bang (with showers) the τ track can be reconstructed or identified and this scenario (the τ track and the second bang) is called the lollipop events. An inverted lollipop event is one where only the first bang ($\nu_\tau + N \rightarrow \tau + X$) is detected and the subsequent τ track is detected or reconstructed. As mentioned in Ref. [6], the detection of ν_τ from their CC interaction mentioned above is not every efficient by a 1km^3 detector since the double bang events can possibly be detected only for the ν_τ energies between 1 PeV to 20 PeV beyond which the tau decay length is longer than the width of such detector and at still higher energies the flux is too small for such detectors for their detection. Hence, in the present work we do not consider the events initiated by $\nu_\tau N$ CC interactions. However, for ν_τ we consider the process that may yield events higher than the "double bang" events. We consider the decay channel of τ lepton [7], obtained from charged current interactions of ν_τ, where muons are produced ($\nu_\tau \rightarrow \tau \rightarrow \bar{\nu}_\mu \mu \nu_\tau$) which can then be detected as muon tracks [8] in ICECUBE detector. The neutral current (NC) interactions of all flavour however will produce the shower events at ICECUBE and they are considered in this investigation.

This paper is organised as follows. In Section 2 we describe the formalism for neutrino fluxes of the three species while reaching the earth. The nature of the GRB flux taken for present calculations is also discussed. The flux suffers flavour oscillations while traversing from GRB site to the earth. The oscillation probabilities are also calculated and the oscillated flux obtained on reaching the earth is determined. They are given in Section 2.1. We also describe in this section the analytical expressions for the yield of secondary muons and shower events at the ice cerenkov kilometre square detector like ICECUBE. This is given in Section 2.2. The actual calculations and results are discussed in Section 3. Finally, in Section 4, some discussions and concluding remarks are given.

2 Formalism

2.1 GRB Neutrinos Fluxes

The neutrino production in GRB is initiated through the process of cosmological beam dump by which a highly accelerated protons from GRB interacts with γ to produce pions which in turn decays to produce $\nu_\mu(\bar{\nu_\mu})$ and $\nu_e(\bar{\nu_e})$ much the same ways as atmospheric neutrinos are produced. They are produced in the proportion $2\nu_\mu : 2\bar{\nu_\mu} : 1\nu_e : 1\bar{\nu_e}$ [9].

For the present calculation we consider the isotropic flux [10] resulting from the summation over the sources and as given in Gandhi et al [11]. The isotropic GRB flux for $\nu_\mu + \bar{\nu_\mu}$ is given as

$$\mathcal{F}(E_\nu) = \frac{dN_{\nu_\mu+\bar{\nu_\mu}}}{dE_\nu} = \mathcal{N}\left(\frac{E_\nu}{1\mathrm{GeV}}\right)^{-n} \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}\mathrm{GeV}^{-1} \quad (1)$$

In the above,

$$\mathcal{N} = 4.0 \times 10^{-13}, \quad n = 1, \quad \text{for } E_\nu < 10^5 \text{ GeV}$$

$$\mathcal{N} = 4.0 \times 10^{-8}, \quad n = 2, \quad \text{for } E_\nu > 10^5 \text{ GeV}$$

Thus,

$$\begin{aligned} \frac{dN_{\nu_\mu}}{dE_\nu} &= \phi_{\nu_\mu} = \frac{dN_{\bar{\nu_\mu}}}{dE_\nu} = \phi_{\bar{\nu_\mu}} = 0.5\mathcal{F}(E_\nu) \\ \frac{dN_{\nu_e}}{dE_\nu} &= \phi_{\nu_e} = \frac{dN_{\bar{\nu_e}}}{dE_\nu} = \phi_{\bar{\nu_e}} = 0.25\mathcal{F}(E_\nu) \end{aligned} \quad (2)$$

The neutrinos undergo flavour oscillation during their passage from the GRB to the earth. Under 3-neutrino oscillation, the ν_e and ν_μ originally created at GRB will be oscillated to nu_τ. Thus after flavour oscillations, the ν_e fluxes (F_{ν_e}), ν_μ fluxes (F_{ν_μ}), ν_τ fluxes (F_{ν_τ}) become

$$\begin{aligned} F_{\nu_e} &= P_{\nu_e\to\nu_e}\phi_{\nu_e} + P_{\nu_\mu\to\nu_e}\phi_{\nu_\mu} \\ F_{\nu_\mu} &= P_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P_{\nu_e\to\nu_\mu}\phi_{\nu_e} \\ F_{\nu_\tau} &= P_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu} \quad . \end{aligned} \quad (3)$$

The transition probability of a neutrino of flavour α to a flavour β is given by,

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha_i} U_{\beta_i} U_{\alpha_j} U_{\beta_j} \sin^2 \left(\frac{\pi L}{\lambda_{ij}} \right) \tag{4}$$

In the above oscillation length λ_{ij} is given by

$$\lambda_{ij} = 2.47 \text{ Km} \left(\frac{E}{\text{GeV}} \right) \left(\frac{\text{eV}^2}{\Delta m^2} \right) \tag{5}$$

Because of astronomical baseline $\Delta m^2 L/E >> 1$, the oscillatory part becomes averaged to half. Thus,

$$\left\langle \sin^2 \left(\frac{\pi L}{\lambda_{ij}} \right) \right\rangle = \frac{1}{2} \tag{6}$$

Therefore

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &= \delta_{\alpha\beta} - 2 \sum_{j>i} U_{\alpha_i} U_{\beta_i} U_{\alpha_j} U_{\beta_j} \\ &= \delta_{\alpha\beta} - \sum_{i} U_{\alpha_i} U_{\beta_i} \left[\sum_{j \neq i} U_{\alpha_j} U_{\beta_j} \right] \\ &= \sum_j |U_{\alpha_j}|^2 |U_{\beta_j}|^2 \end{aligned} \tag{7}$$

where use has been made of the condition $\sum_i U_{\alpha_i} U_{\beta_i} = \delta_{\alpha\beta}$.

With Eq. (7), Eq. (3) can be rewritten in matrix form

$$\begin{aligned} \begin{pmatrix} F_{\nu_e} \\ F_{\nu_\mu} \\ F_{\nu_\tau} \end{pmatrix} &= \begin{pmatrix} U_{e1}^2 & U_{e2}^2 & U_{e3}^2 \\ U_{\mu1}^2 & U_{\mu2}^2 & U_{\mu3}^2 \\ U_{\tau1}^2 & U_{\tau2}^2 & U_{\tau3}^2 \end{pmatrix} \begin{pmatrix} U_{e1}^2 & U_{\mu1}^2 & U_{\tau1}^2 \\ U_{e2}^2 & U_{\mu2}^2 & U_{\tau2}^2 \\ U_{e3}^2 & U_{\mu2}^2 & U_{\tau3}^2 \end{pmatrix} \begin{pmatrix} \phi_{\nu_e} \\ \phi_{\nu_\mu} \\ \phi_{\nu_\tau} \end{pmatrix} \\ &= \begin{pmatrix} U_{e1}^2 & U_{e2}^2 & U_{e3}^2 \\ U_{\mu1}^2 & U_{\mu2}^2 & U_{\mu3}^2 \\ U_{\tau1}^2 & U_{\tau2}^2 & U_{\tau3}^2 \end{pmatrix} \begin{pmatrix} U_{e1}^2 & U_{\mu1}^2 & U_{\tau1}^2 \\ U_{e2}^2 & U_{\mu2}^2 & U_{\tau2}^2 \\ U_{e3}^2 & U_{\mu2}^2 & U_{\tau3}^2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e} \end{aligned} \tag{8}$$

In Eq. (8) above, we have used the initial flux ratio from GRB to be $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = 1 : 2 : 0$. From Eq. (8) it then follows that,

$$\begin{aligned}
F_{\nu_e} &= \Big\{U_{e1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{e2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] + \\
&\quad U_{e3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\Big\}\,\phi_{\nu_e} \\
F_{\nu_\mu} &= \Big\{U_{\mu 1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{\mu 2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] + \\
&\quad U_{\mu 3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\Big\}\,\phi_{\nu_e} \\
F_{\nu_\tau} &= \Big\{U_{\tau 1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{\tau 2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] + \\
&\quad U_{\tau 3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\Big\}\,\phi_{\nu_e} \qquad (9)
\end{aligned}$$

The MNS mixing matrix U for 3-flavour case is given as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix} \qquad (10)$$

We are not considering any CP violation here. Hence Eq. (3) - (9) above holds also for antineutrinos.

2.2 Detection of GRB neutrinos

The ν_μ's from a GRB can be detected from the tracks of the secondary muons produced through the ν_μ CC interactions.

The total number of secondary muons induced by GRB neutrinos at a detector of unit area is given by (following [12, 9, 13])

$$S = \int_{E_{\rm thr}}^{E_{\nu \rm max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm surv}(E_\nu) P_\mu(E_\nu, E_{\rm thr}) \qquad (11)$$

In the above, $P_{\rm surv}$ is the probability that a neutrino reaches the detector without being absorbed by the earth. This is a function of the neutrino-nucleon interaction length in the earth and the effective path length $X(\theta_z)$

(gm cm^{-2}) for incident neutrino zenith angle θ_z ($\theta_z = 0$ for vertically downward entry with respect to the detector). This attenuation of neutrinos due to passage through the earth is referred to as shadow factor. For an isotropic distribution of flux, this shadow factor (for upward going neutrinos) is given by

$$P_{\rm surv}(E_\nu) = \frac{1}{2\pi}\int_{-1}^{0} d\ \cos\theta \int d\phi \exp[-X(\theta_z)/L_{\rm int}]. \tag{12}$$

where interaction length $L_{\rm int}$ is given by

$$L_{\rm int} = \frac{1}{\sigma^{\rm tot}(E_\nu) N_A} \tag{13}$$

In the above $N_A (= 6.022 \times 10^{23} {\rm gm}^{-1})$ is the Avogadro number and $\sigma^{\rm tot} (= \sigma^{\rm CC} + \sigma^{\rm NC})$ is the total cross section. The effective path length $X(\theta_z)$ is calculated as

$$X(\theta_z) = \int \rho(r(\theta_z, \ell) d\ell. \tag{14}$$

In Eq. (9), $\rho(r(\theta_z, \ell)$ is the matter density inside the earth at a distance r from the centre of the earth for neutrino path length ℓ entering into the earth with a zenith angle θ_z. The quantity $P_\mu(E_\nu, E_{\rm thr})$ in Eq. (6) is the probability that a secondary muon is produced by CC interaction of ν_μ and reach the detector above the threshold energy $E_{\rm thr}$. This is then a function of $\nu_\mu N$ (N represents nucleon) - CC interaction cross section $\sigma^{\rm CC}$ and the range of the muon inside the rock.

$$P_\mu(E_\nu, E_{\rm thr}) = N_A \sigma^{\rm CC} \langle R(E_\nu; E_{\rm thr})\rangle \tag{15}$$

In the above $\langle R(E_\nu; E_{\rm thr})\rangle$ is the average muon range given by

$$\langle R(E_\nu; E_{\rm thr})\rangle = \frac{1}{\sigma^{\rm CC}} \int_0^{1-E_{\rm thr}/E_\nu} dy R(E_\nu(1-y), E_{\rm thr}) \frac{d\sigma^{\rm CC}(E_\nu, y)}{dy} \tag{16}$$

where $y = (E_\nu - E_\mu)/E_\nu$ is the fraction of energy loss by a neutrino of energy E_ν in the charged current production of a secondary muon of energy E_μ. Needless to say that a muon thus produced from a neutrino with energy

E_ν can have the detectable energy range between $E_{\rm thr}$ and E_ν. The range $R(E_\mu, E_{\rm thr})$ for a muon of energy E_μ is given as

$$R(E_\mu, E_{\rm thr}) = \int_{E_{\rm thr}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \simeq \frac{1}{\beta} \ln\left(\frac{\alpha + \beta E_\mu}{\alpha + \beta E_{\rm thr}}.\right) \tag{17}$$

The average lepton energy loss with energy E_μ per unit distance travelled is given by [12]

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu \tag{18}$$

The values of α and β used in the present calculations are

$$\begin{aligned} \alpha &= \{2.033 + 0.077 \ln[E_\mu({\rm GeV})]\} \times 10^{-3} {\rm GeV cm^2 gm^{-1}} \\ \beta &= \{2.033 + 0.077 \ln[E_\mu({\rm GeV})]\} \times 10^{-6} {\rm cm^2 gm^{-1}} \end{aligned} \tag{19}$$

for $E_\mu \lesssim 10^6$ GeV [14] and

$$\begin{aligned} \alpha &= 2.033 \times 10^{-3} {\rm GeV cm^2 gm^{-1}} \\ \beta &= 3.9 \times 10^{-6} {\rm cm^2 gm^{-1}} \end{aligned} \tag{20}$$

otherwise [15]. For muon events obtained from $\nu_m u$ CC interactions, $\frac{dN_\nu}{dE_\nu}$ in Eq. (11) will be replaced by F_{ν_τ} (Eq. 9).

As discussed earlier, the events due to ν_τ CC interactions is considered only for the process where the decay of secondary τ lepton produces muon which then detected by the muon track. The probability of production of muons in the decay channel $\tau \rightarrow \bar{n}u_\mu \mu \nu_\tau$ is 0.18 [7, 8]. The generated muon carries a fraction 0.3 of energy of original ν_τ (a fraction 0.75 of the energy of the $\nu_t au$ is carried by secondary τ lepton and a fraction of 0.4 of τ lepton energy is carried by the muon [7, 12, 8]). For the detection of such muons, the Eqs. (10 - 16) is applicable with properly incorporating the muon energy described above. Needless to say, in this case, $\frac{dN_\nu}{dE_\nu}$ in Eq. (11) is to be replaced by F_{ν_τ} (Eq. 9).

For the case of showers, we do not have the advantage of a specific track and then the whole detector volume is to be considered. The event rate for the shower case is given by

$$N_{\rm sh} = \int dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm surv}(E_\nu) \times \int \frac{1}{\sigma^j} \frac{d\sigma^j}{dy} P_{\rm int}(E_\nu, y) \ . \tag{21}$$

In the above, $\sigma^j = \sigma^{\rm CC}$ (for electromagnetic shower from ν_e charged current interactions or $\sigma^{\rm NC}$ as the case may be. In the above $P_{\rm int}$ is the probability that a shower produced by the neutrino interactions will be detected and is given by

$$P_{\rm sh} = \rho N_A \sigma^j L \tag{22}$$

where ρ is the density of the detector material and L is the length of the detector (L = 1 Km for ICECUBE).

For each case of shower events, $\frac{dN_\nu}{dE_\nu}$ in Eq. (21) is to be replaced by F_{ν_e} or F_{ν_μ} or F_{ν_τ} as the case may be.

3 Calculations and Results

The secondary muon yield at a kilometre scale detector such as ICECUBE is calculated using Eqs. (6 - 15). The earth matter density in Eq. (9) is taken from [9] following the Preliminary Earth Reference Model (PREM). The interaction cross-sections - both charged current and total - used in these equations are taken from the tabulated values (and the analytical form) given in Ref. [11]. In the present calculations $E_{\nu{\rm max}} = 10^{11}$ GeV and threshold energy $E_{\rm thr} = 1$ TeV are considered.

For our investigations, we first define a ratio R of the muon events (both from ν_μ (and $\bar{\nu}_\mu$) and ν_τ (and $\bar{\nu}_\tau$)) and the shower events. As described in the previous sections, the muon events are from ν_μ (and $\bar{\nu}_\mu$) and ν_τ (and $\bar{\nu}_\tau$), whereas the shower events include electromagnetic shower initiated by CC interaction of ν_e and NC interactions of neutrinos of all flavours. Therefore,

$$\mathcal{R} = \frac{T_\mu}{T_{\rm sh}} \tag{23}$$

where,

$$\begin{aligned} T_\mu &= S(\text{for}\ \nu_\mu) + S(\text{for}\ \nu_\tau) \\ T_{\rm sh} &= N_{\rm sh}(\text{for}\ \nu_e\ \text{CC interaction}) \\ &+ N_{\rm sh}(\text{for}\ \nu_e\ \text{NC interaction}) \\ &+ N_{\rm sh}(\text{for}\ \nu_\mu\ \text{NC interaction}) \\ &+ N_{\rm sh}(\text{for}\ \nu_\tau\ \text{NC interaction}) \end{aligned} \tag{24}$$

The purpose of this work is to explore whether UHE neutrinos from GRB will be able to distinguish any variation of θ_{12} and θ_{23} from their best fit values. The tri-bimaximal mixing condition is denoted by the best fit values of θ_{12} and θ_{23} for $\theta_{13} = 0^o$. The best fit value of $\theta_{12} = 35.2^o$ and that of $\theta_{23} = 45^o$. We first vary θ_{12} in the limit $30^o \leq \theta_{12} \leq 38^o$ and vary θ_{23} in the limit $38^o \leq \theta_{12} \leq 54^o$ $\theta_{13} = 0$ and for each case calculate the ratio $\mathcal{R}$ using Eqs. (1 - 24). We find that $\mathcal{R}$ varies from 3.14 to 4.25. One readily sees that the variation in muon to shower ratio is not very significant. The flux and other uncertainties of the detector may wash away this small variations. $\mathcal{R}$ obtained from tri-bimaximal condition given above is 4.05.

The same operation is repeated for three different values of $\theta_1 3$, namely $\theta_1 3 = 3^o$ 6^o and 9^o with similar results. The results are tabulated below.

θ_{13}	$\mathcal{R}_{\rm Max}$	$\mathcal{R}_{\rm Min}$	$\mathcal{R}$ at $\theta_{12} = 35.2^o$, $\theta_{23} = 45^o$
0^o	4.78	3.80	4.05
3^o	4.75	3.77	4.01
6^o	4.72	3.75	3.98
9^o	4.69	3.73	3.96

Table 1. Maximum and minimum values of ratio $\mathcal{R}$ for different values of mixing angles

As is evident from Table 1, the variation of shower ratio is not very significant with the deviation from the best fit values of the mixing angles. Therefore it is diffcult by a detector like ICECUBE to detect the deviattion, if any, from tri-bimaximal mixing through the detection of UHE neutrinos from a GRB.

4 Discussions and conclusions

In summary, we investigate the deviation from the well known tri-bimaximal mixing in the case of Ultra High Energy neutrinos from a Gamma Ray Burst detected in a kilometer scale detector such as ICECUBE. We have calculated the ratio $\mathcal{R}$ of the muon track events and shower events (electromagnetic shower from charged current interactions of ν_e and hadronic showers from neutral current interactions of neutrinos of all flavours) for tri-bimaximal mixing given by $\theta_{12} = 35.2^o$, $\theta_{23} = 35.2^o$, $\theta_{13} = 0^o$. We then investigate the posiibile variation of $\mathcal{R}$ from tri-bimaximal mixing condition by varying θ_{12} and θ_{23} within their experimentally obtained range for four different values of θ_{13} namely 0^o, 3^o 6^o and 9^o. The variation in $\mathcal{R}$ is small ($\sim 8\%$ to $\sim 18\%$). In order to detect this variation with a detector like ICECUBE the precision level is to be high and enough exposure may be required. We also want to emphasis here we have repeated the same calculation for single GRBs with fixed red shift (z) values with similar results.

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